Study Guide Solutions - Quadratics

1. The equation of a curve is $y^2+2x=13$ and the equation of a line is $2y+x=k$ where k is a constant.

i) In the case of $k=8$, find the coordinates of the points of the intersection of the line and the curve.

$y^2+2x=13$

$2x=13-y^2$

$\boxed{x=\frac{13-y^2}{2}}$

$2y+x=8$

$\boxed{x=8-2y}$

$\cancel{2\cdot}\frac{13-y^2}{\cancel{2}}=(8-2y)\cdot 2$

$13-y^2=2(8-2y)$

$13-y^2=(16-4y)$

$0=y^2-4y-13+16$

$0=y^2-4y+3$

$(y-3)(y-1)$

$y=3$ and $y=1$

$x=8-2(3)$

$x=8-6=2$

$x=8-2(1)$

$x=8-2$

$x=6$

$\boxed{(2,3)(6,1)}$

ii) Find the value of k which the line is tangent to the curve.

$x=\frac{13-y^2}{2}$ $\quad x=k-2y$

$\frac{13-y^2}{2}=k-2y$

$13-y^2=2k-4y$

$y^2-4y-13+2k=0$

$\left(\frac{-4}{2}\right)^2=$ $\quad y^2-4y+4$

Perfect squares have 1 point (tangent!)

$-13+2k=4$
$+13 \qquad +13$

$\frac{2k}{2}=\frac{17}{2}$

$\boxed{k=\frac{17}{2}}$

2. i) Express $2x^2-4x+1$ in the form $a(x+b)^2+c$ and hence state the coordinates of the minimum point A on the curve $y=2x^2-4x+1$.

$$2x^2-4x \quad +1=0$$

$$2(x^2-2x+1)+1\underset{-2}{=}0\underset{-2}{+2}$$

$$\boxed{2(x-1)^2-1}=0$$

$$\boxed{\text{minimum } (1,-1)}$$

3. A line has an equation $y=kx+6$ and a curve has an equation $y=x^2+3x+2k$ where k is a constant.

i) For the case where $k=2$, the line and the curve intersect at points A and B. Find the distance AB and the coordinates of the midpoint of AB.

$$y=x^2+3x+2(2)$$

$$y=2x+6$$

$$2x+6=x^2+3x+4$$

$$-2x \quad -6 \qquad -2x \quad -6$$

$$x^2+x-2=0$$

$$(x+2)(x-1)=0$$

$$x=-2,1$$

$$2(-2)+6=2$$

$$2(1)+6=8$$

$$\boxed{\begin{matrix}(-2,2) & (1,8)\\ \text{Point A} & \text{Point B}\end{matrix}}$$

$$\text{midpoint}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{-2+1}{2},\frac{2+8}{2}\right)$$

$$\boxed{\text{midpoint of AB}=\left(-\tfrac{1}{2},5\right)}$$

$$\text{distance: } \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\sqrt{(1--2)^2+(8-2)^2}$$

$$\sqrt{3^2+6^2}$$

$$\boxed{\sqrt{45}=6.708 \quad \text{distance}=}$$

3ii) Find the two values of k for which the line is a tangent to the curve.

$y = kx + 6 \qquad y = x^2 + 3x + 2k$

$kx + 6 = x^2 + 3x + 2k$

$x^2 + 3x - kx + 2k - 6 = 0$

$x^2 + (3-k)x + 2k - 6 = 0$

$(3-k)^2 - 4(1)(2k-6) = 0$

$k^2 - 6k + 9 - 8k + 24 = 0$

$k^2 - 14k + 33 = 0$

$(k-11)(k-3) = 0$

$\boxed{k = 11,\ k = 3}$

* tangent has one point of intersection so the discriminant is zero

4 Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points.

$y = 2x - k \qquad y = x^2 + kx - 2$

$2x - k = x^2 + kx - 2$

$x^2 - 2x + kx + k - 2 = 0$

$x^2 + (-2 + k)x + k - 2 = 0$

$(-2+k)^2 - 4(1)(k-2) > 0$

$k^2 - 4k + 4 - 4k + 8 > 0$

$k^2 - 8k + 12 > 0$

$(k-6)(k-2) > 0$

$k > 6 \quad k < 2$

$\boxed{k < 2,\ k > 6}$

* Discriminant > 0 if there are two distinct points

5 A curve has an equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$ where m is constant.

i) For the case where $m = 1$, the curve and the line intersect at points A and B. Find the coordinates of the midpoint of AB.

$y = x^2 - 4x + 4$ $\quad y = 1x$

$x^2 - 4x + 4 = 1x$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4,\ x = 1$

$(4,4)$

$(1,1)$

midpoint: $\left(\frac{1+4}{2}, \frac{1+4}{2}\right)$

$\boxed{\text{midpoint: } \left(\frac{5}{2}, \frac{5}{2}\right)}$

ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve.

$y = x^2 - 4x + 4$ $\quad y = mx$

$x^2 - 4x + 4 = mx$

$x^2 - 4x - mx + 4 = 0$

$x^2 + (-4 - m)x + 4 = 0$

$(-4-m)^2 - 4(1)(4) = 0$

$m^2 + 8m + 16 - 16 = 0$

$m^2 + 8m = 0$

$m(m+8) = 0$

$\boxed{m = -8}$

*Tangent → discriminant is zero!

$y = x^2 - 4x + 4$ $\quad y = -8x$

$x^2 - 4x + 4 = -8x$

$x^2 + 4x + 4 = 0$

$(x+2)(x+2) = 0$

$x = -2$

$y = -8(-2) \rightarrow 16$

$\boxed{\text{Point of tangent: } (-2, 16)}$

6. Functions f and g are defined for $x \in R$ by

$f: x \to 2x+1$

$g: x \to x^2-2$

i) Find and simplify expressions for $fg(x)$ and $gf(x)$

$fg(x) \to 2(x^2-2)+1$

$fg(x) \to 2x^2-4+1$

$\boxed{fg(x) = 2x^2-3}$

$gf(x) = (2x+1)^2-2$

$gf(x) = 4x^2+4x+1-2$

$\boxed{gf(x) = 4x^2+4x-1}$

ii) Hence find the value of a for which $fg(a) = gf(a)$

$2x^2-3 = 4x^2+4x-1$

$-2x^2 + 3 \quad -2x^2 \quad +3$

$0 = 2x^2+4x+2$

$0 = 2(x^2+2x+1)$

$2(x+1)(x+1)$

$\boxed{a=-1}$

~~IX~~) The line $x-y+4=0$ intersects the curve $y=2x^2-4x+1$ at points P and Q. It is given that the coordinates of P are (3,7).

ii) Find the coordinates of Q.

$x-y+4=0$

$x+4=y$

$y=2x^2-4x+1$

P:(3,7)

$\boxed{Q: (-1/2, 3 1/2)}$

$x+4 = 2x^2-4x+1$

$-x-4 \quad -x \quad -4$

$0=2x^2-5x-3$

$0=x^2-5x-6$

$0=(x-\frac{6}{2})(x+\frac{1}{2})$

$x=3, -1/2$

~~iii) Find the equation of the line joining Q to the midpoint of AP.~~

~~A is (1, 4)~~